3D Gaussian Splatting for Real-Time Radiance Field Rendering

Bernard Kerbl et al.
Presented by: Paolo Didier Alfano

Representing scenes

Representing scenes

Input Image

Representing scenes

Input Image

Train

Representing scenes

Input Image

Train

New views!

Amazing motivational slide

Amazing motivational slide

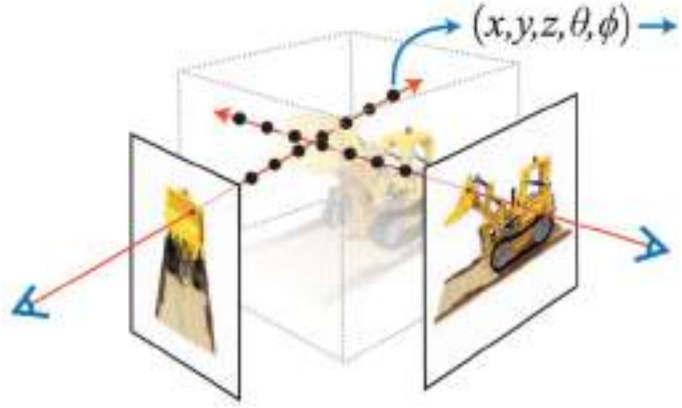
UniGe | MaLGa

NeRF recap

Gaussian splatting

Conclusions

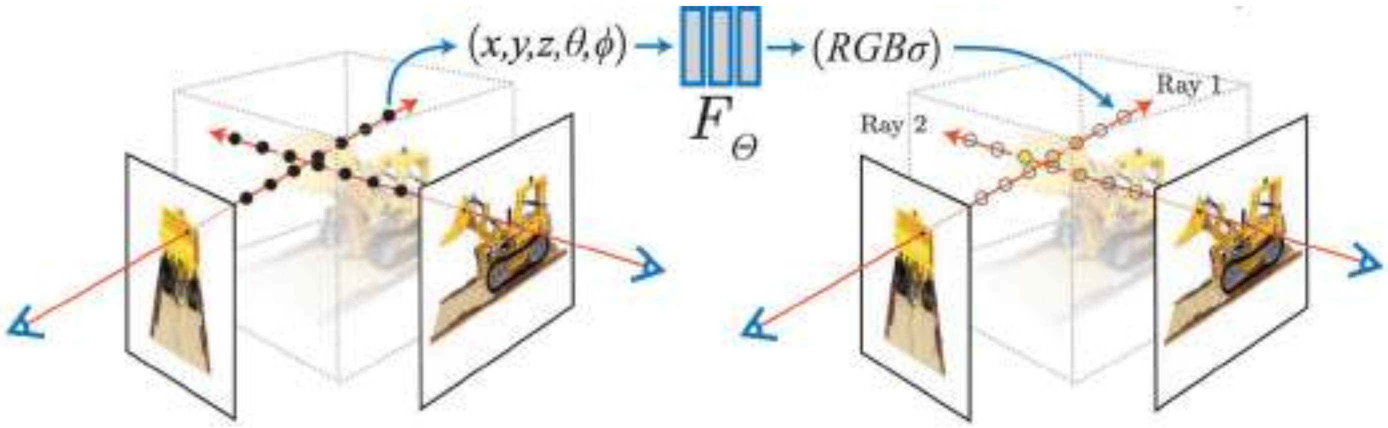
Pipeline: bird's eye view



a) NN input:

point from ray

Pipeline: bird's eye view



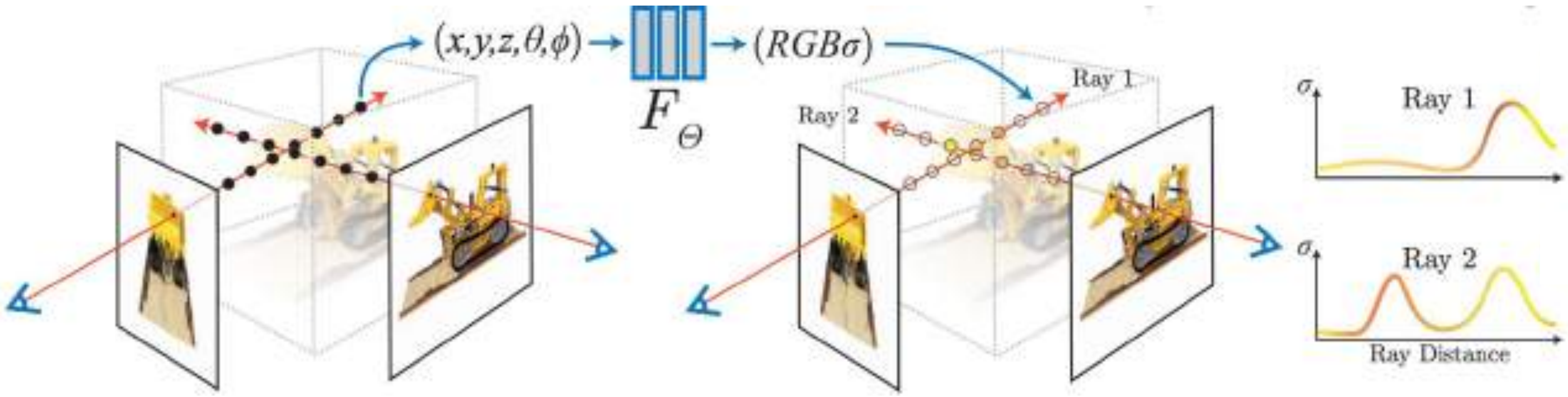
a) NN input:

point from ray

b) NN output:

Point color
and density

Pipeline: bird's eye view



a) NN input:

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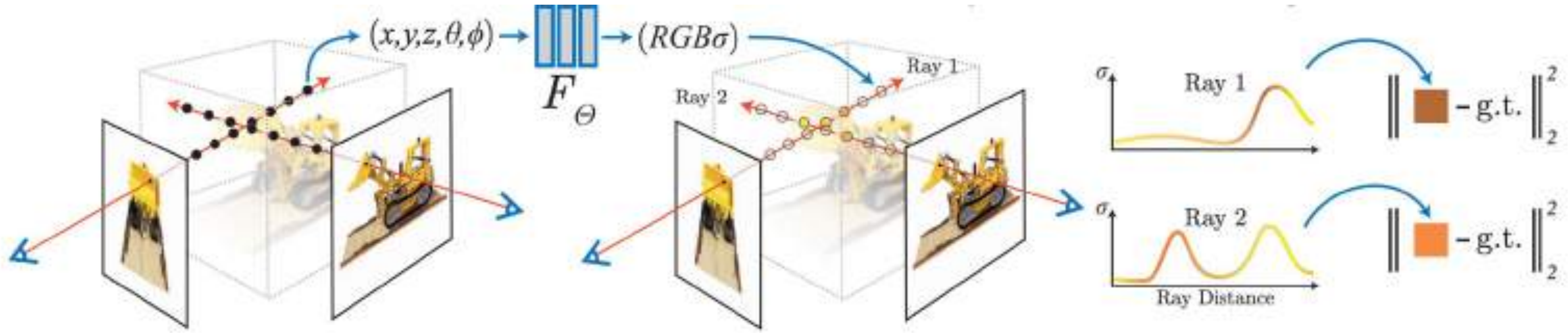
b) NN output:

Point color and density

c) Overall ray

Putting points together

Pipeline: bird's eye view



a) NN input:

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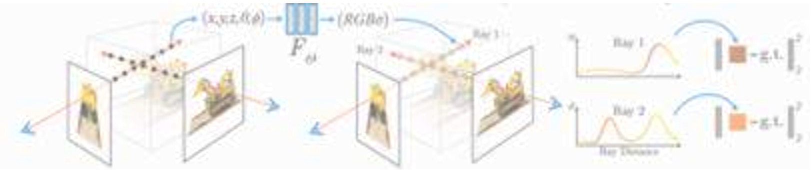
Putting points together

d) Loss

Images & rays



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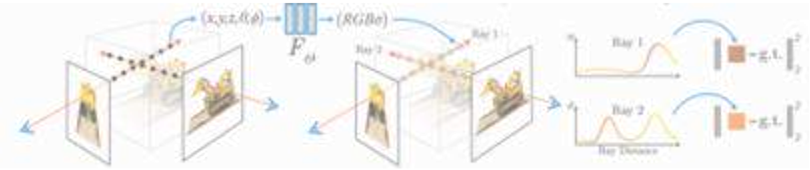
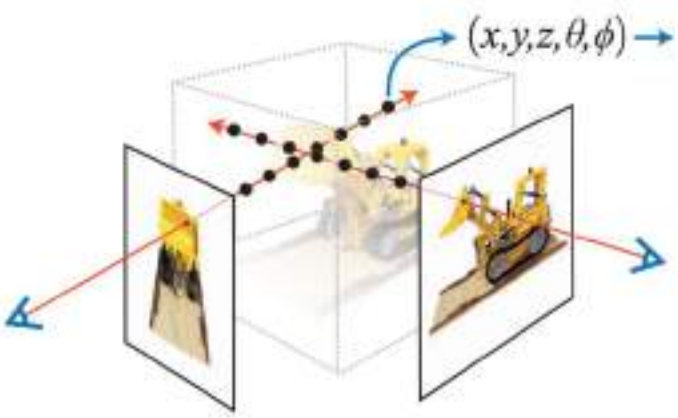


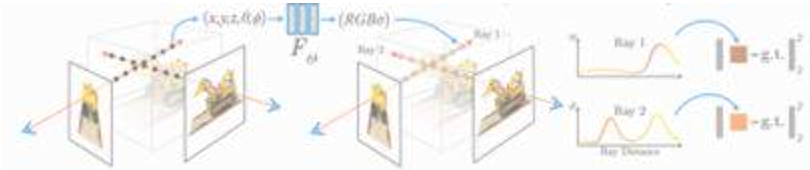


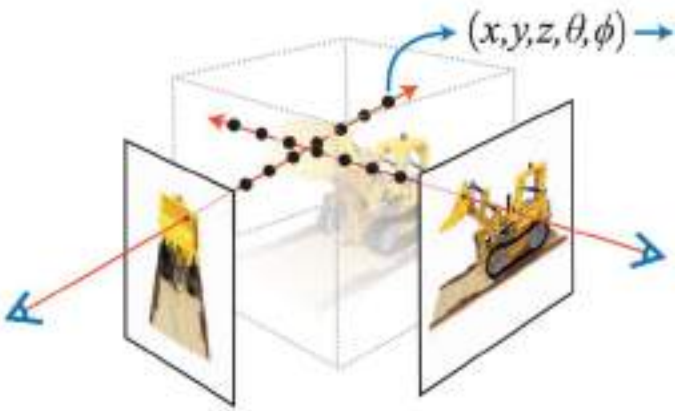
We have:

Images

Camera pose

Images & rays





We have:

Images

Camera pose

Dataset size? $\#\text{images} \cdot \#\text{pixels} \cdot \#\text{points}$

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NeRF recap

Gaussian splatting

Conclusions

Problems with NeRF?

Rendering efficiency?

NeRF: 0.071 fps

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Common problem:

Problems with NeRF?

Rendering efficiency?

NeRF:	0.071 fps

Common problem:

InstantNGP	9.2 fps
Plenoxels	8.2 fps

Problems with NeRF?

Training efficiency?

NeRF:	48 hrs

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Why? $\#\text{images} \cdot \#\text{pixels} \cdot \#\text{points}$

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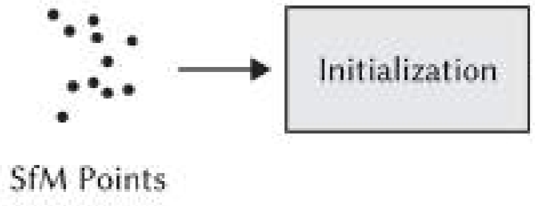
Why? $\#\text{images} \cdot \#\text{pixels} \cdot \#\text{points}$

NN inference, for every sample

Gaussian splatting pipeline



Gaussian splatting pipeline

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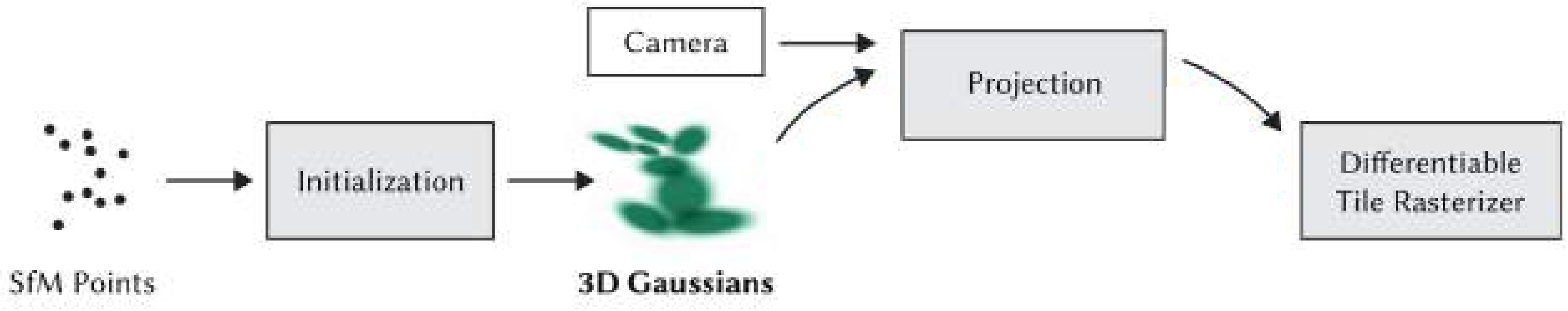

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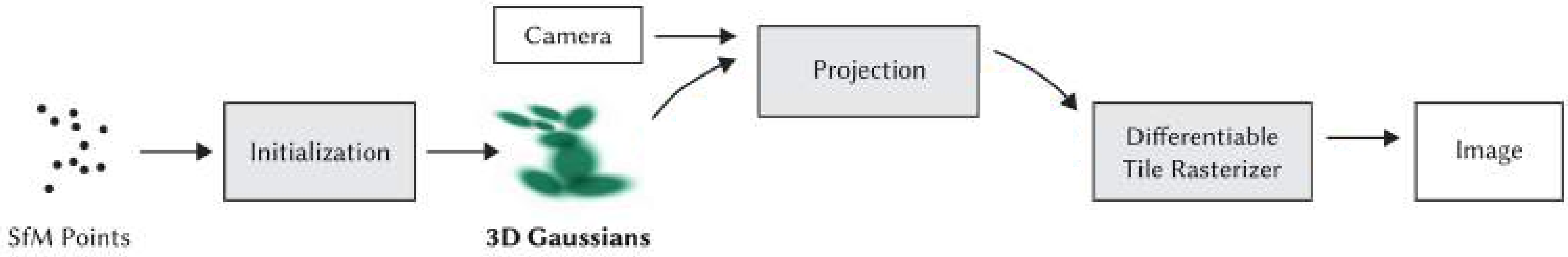

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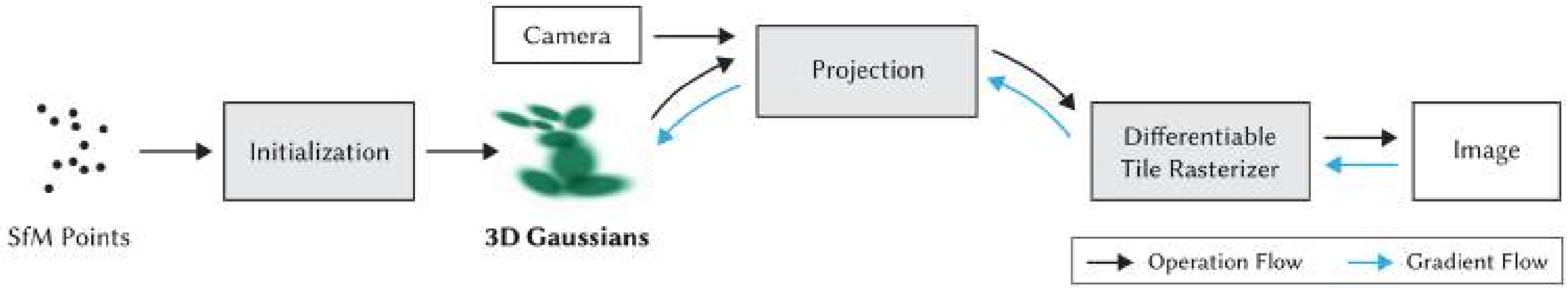

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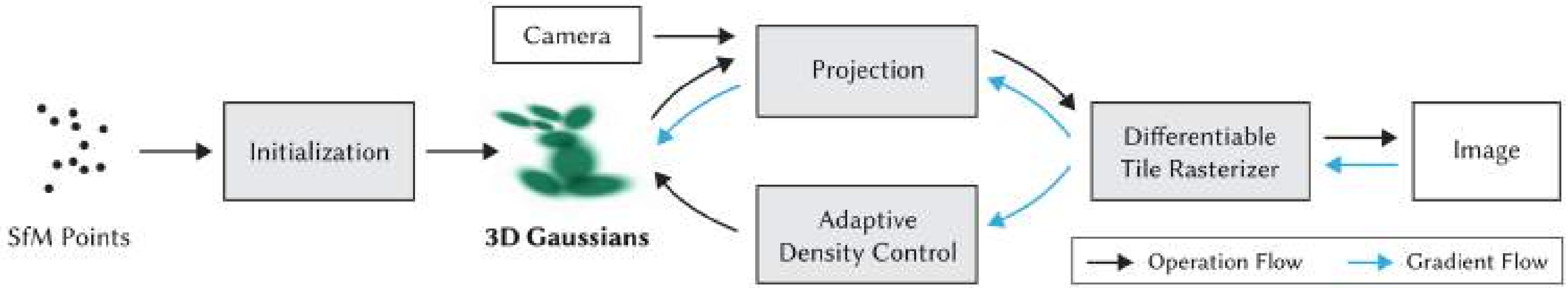

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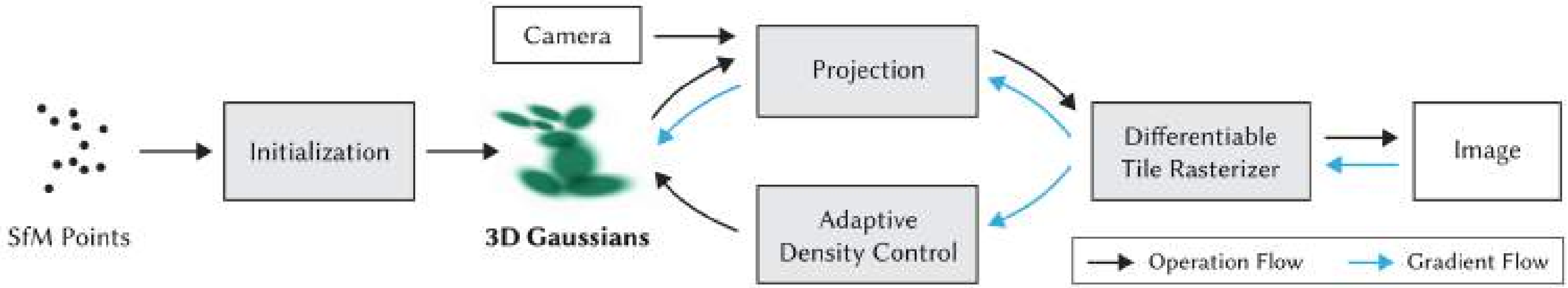
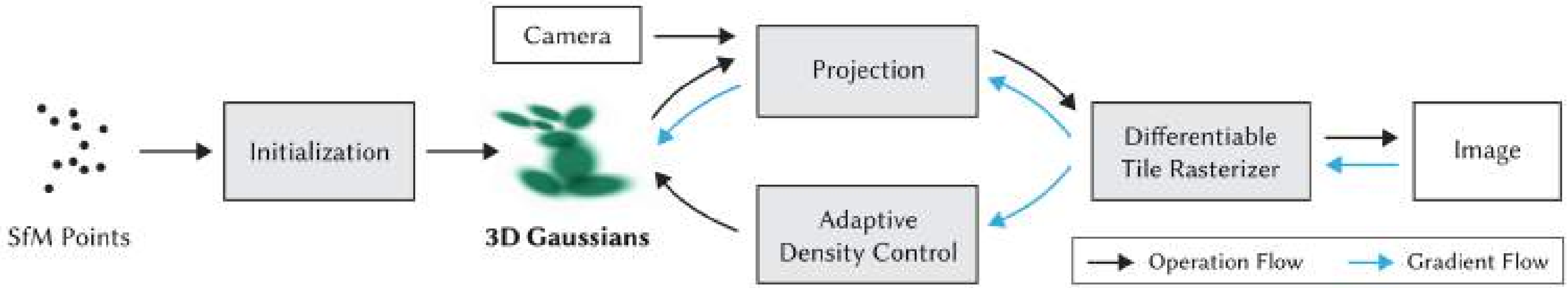


No neural networks!

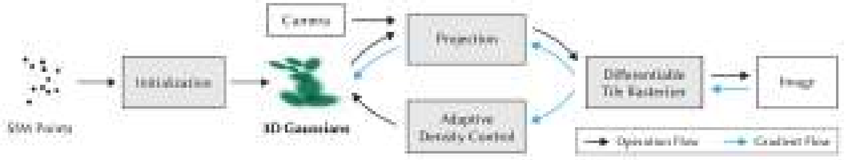
Gaussian splatting pipeline



No neural networks!

Modern ML + traditional computer vision methods

Gaussian splatting pipeline



Structure From Motion (SfM)

Input:

Multiple images

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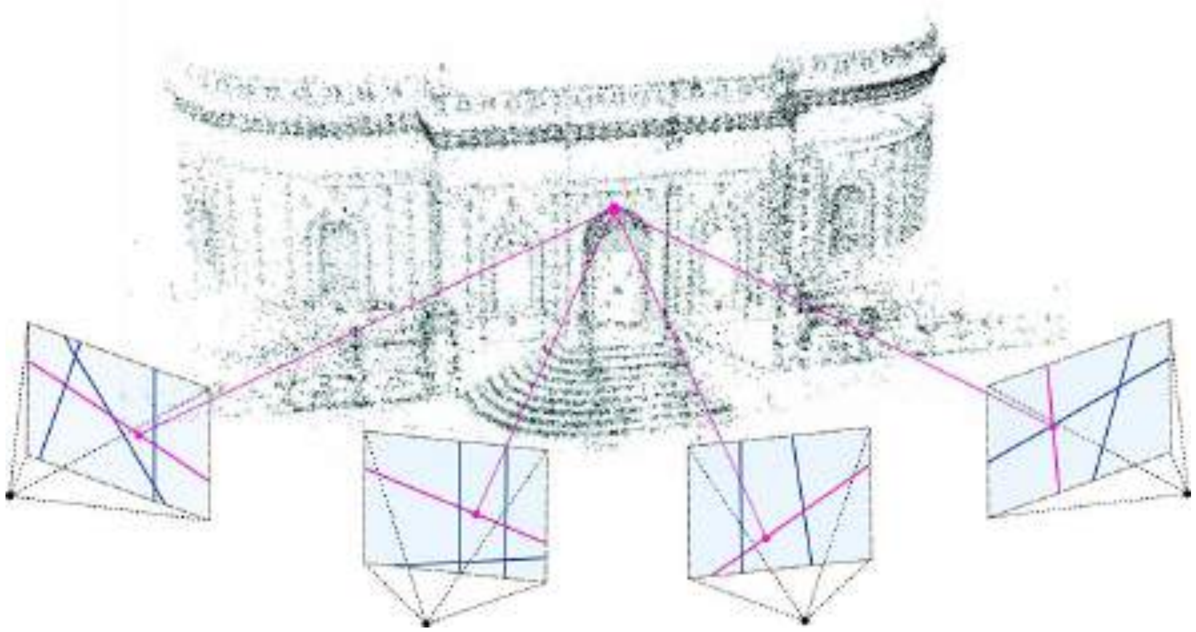
Multiple images

Output:

Reconstruct camera poses



Structure From Motion (SfM)

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Multiple images

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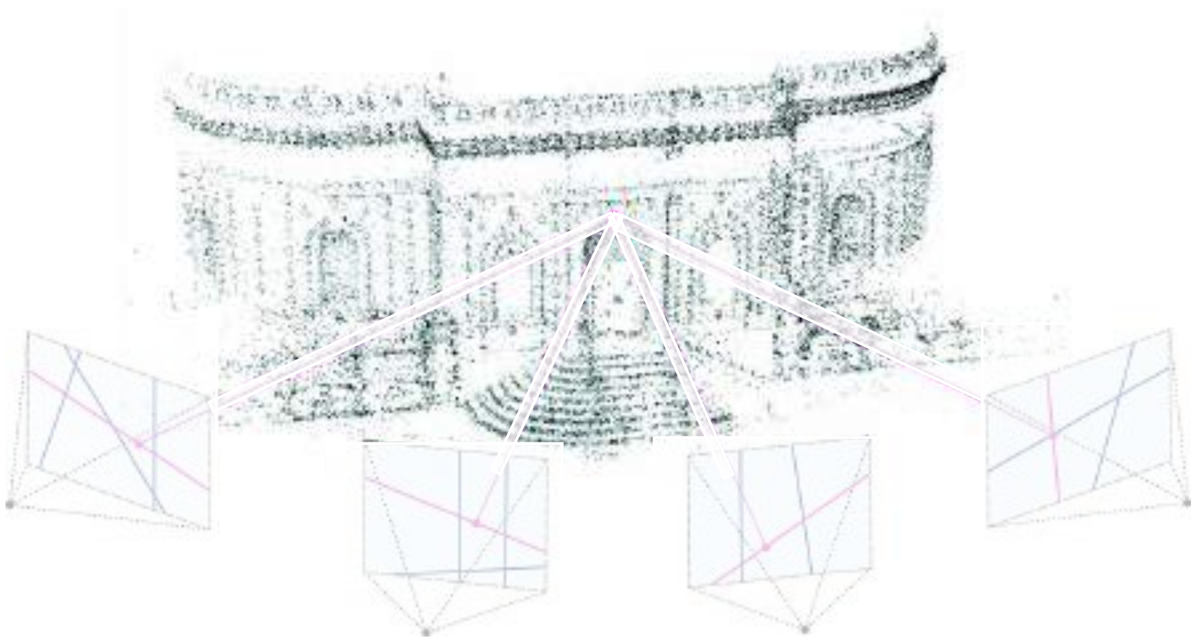
Reconstruct camera poses

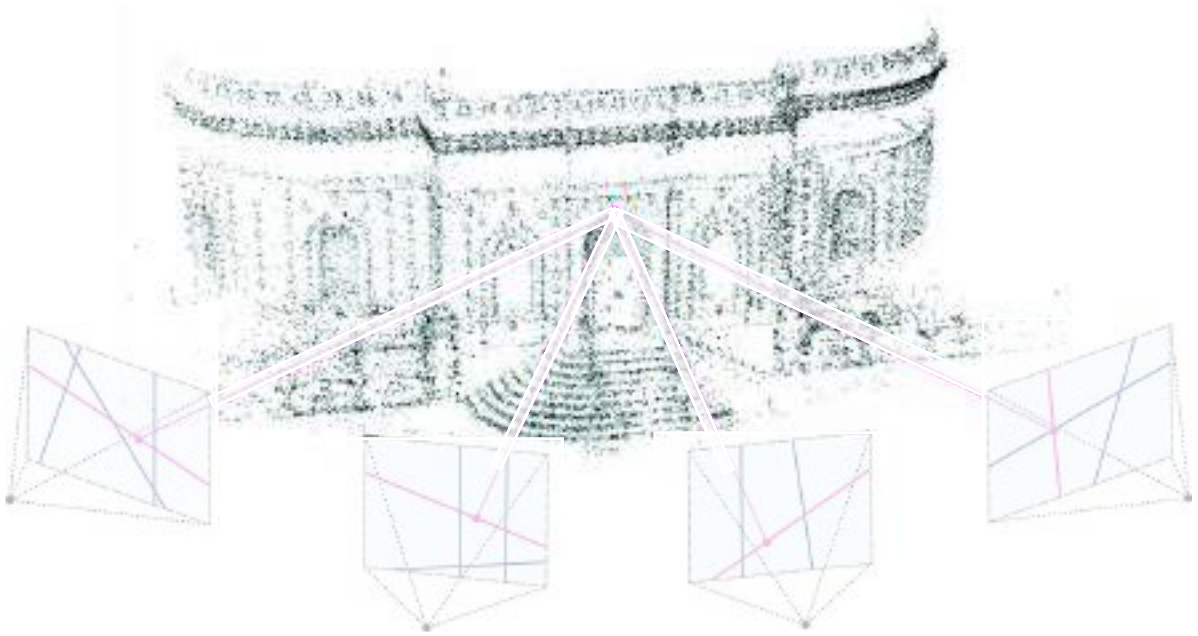
Side effect:

3D sparse point cloud



Structure From Motion (SfM)

Good for initialization



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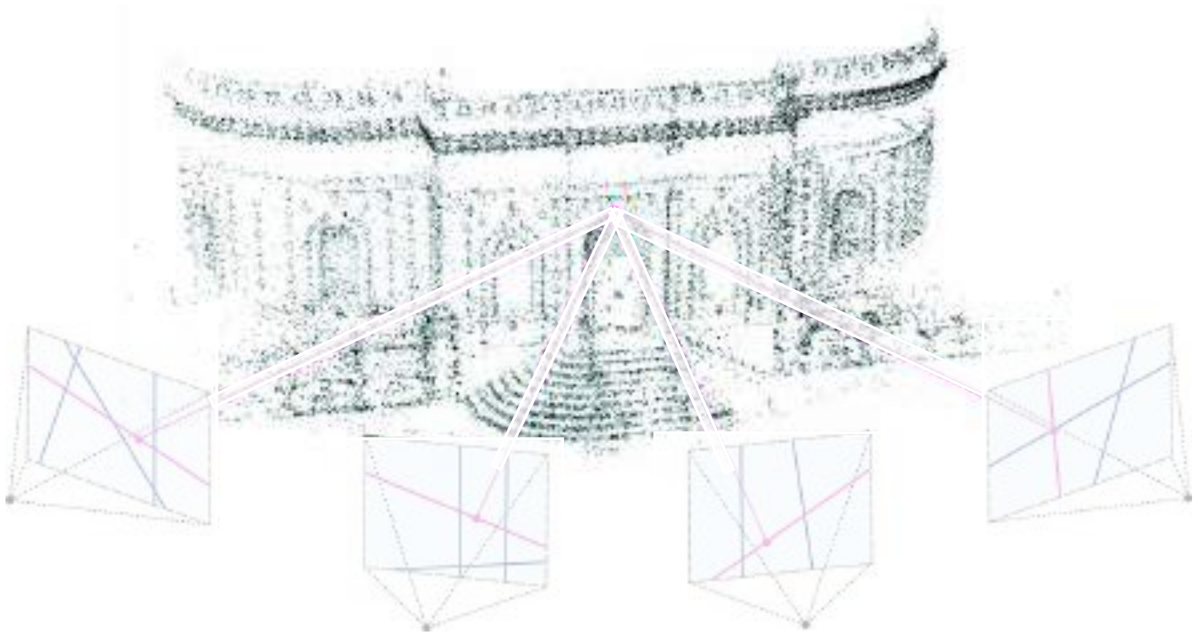
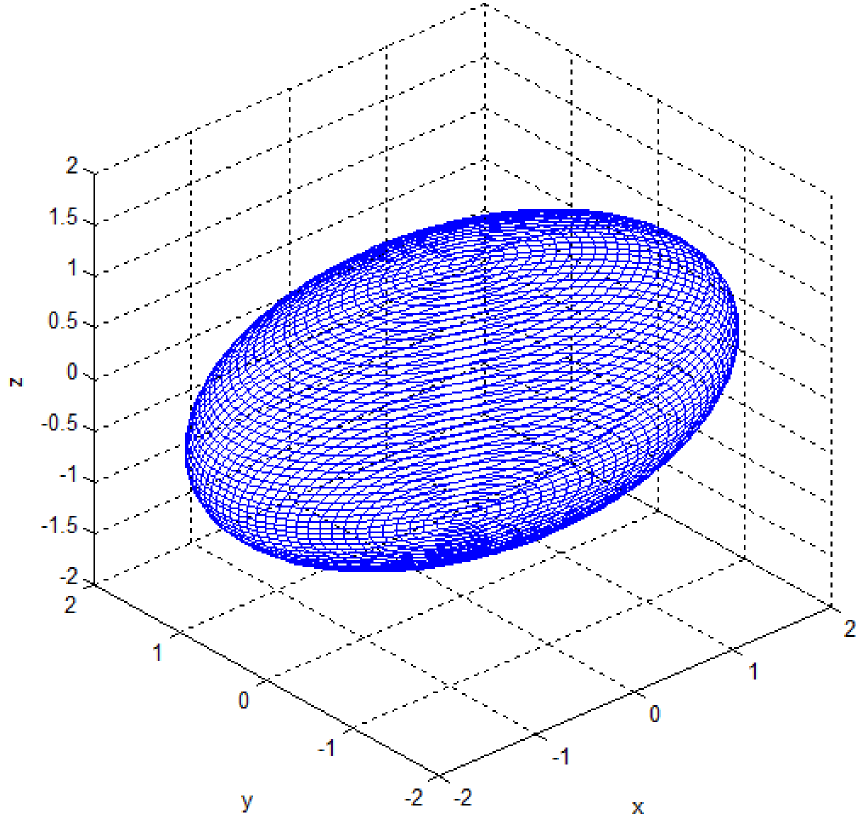


If no point cloud:

- Random initialization
- Worse loss values

What's a 3D gaussian?

Gaussian resembling an ellipsoid



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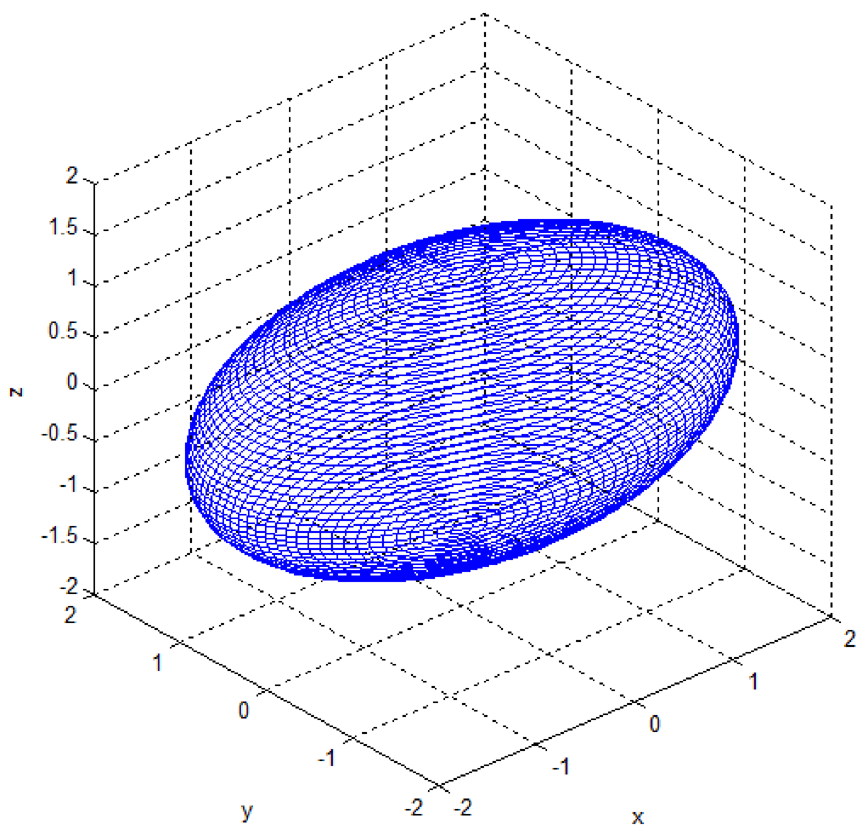
Defined by

Position: X, Y, Z



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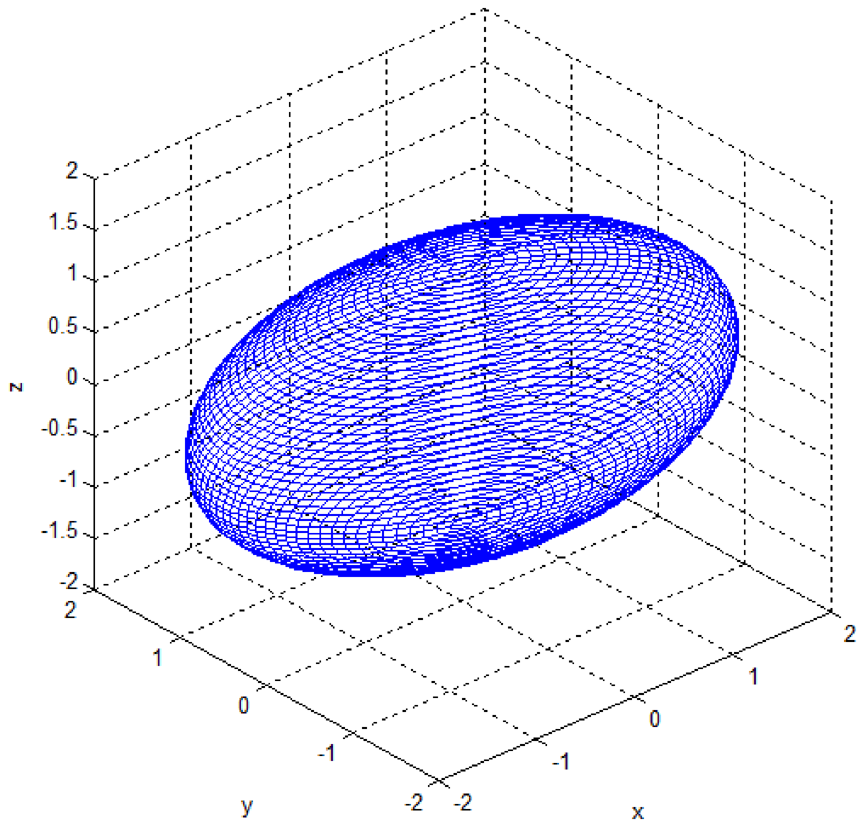

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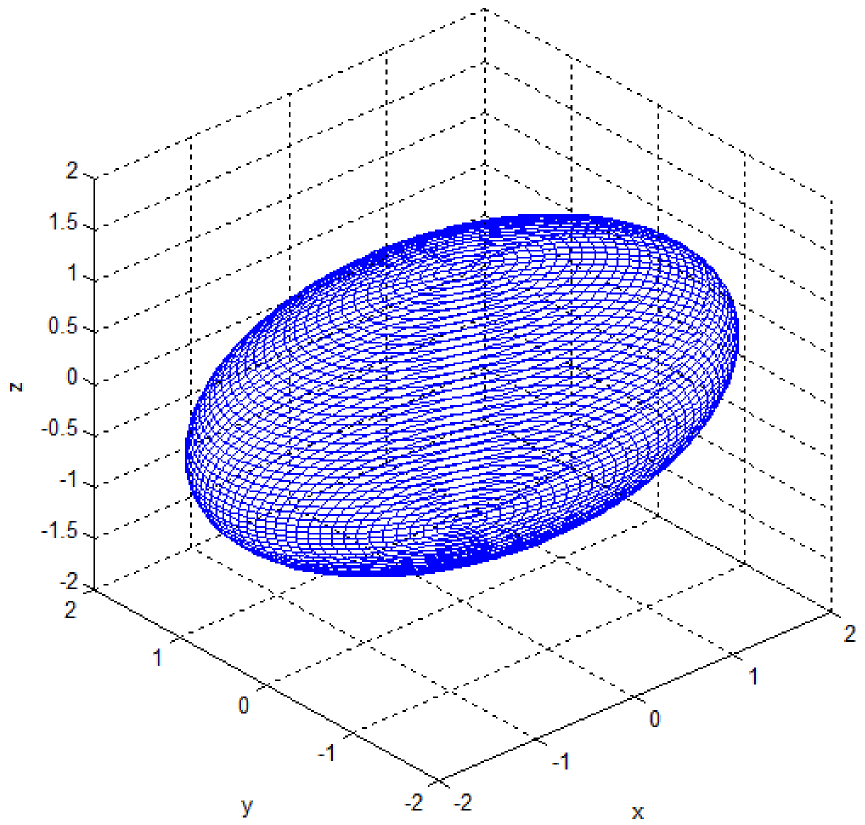

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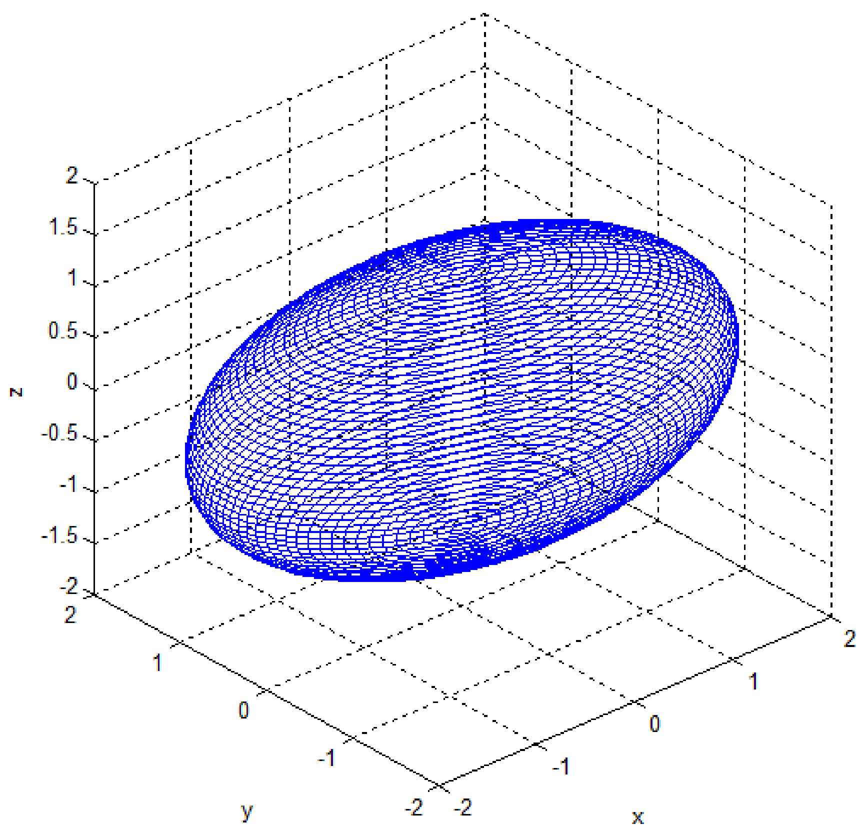
Defined by

- Position: X, Y, Z
- Covariance: 3x3 matrix
- Color: RGB
- Alpha transparency: α



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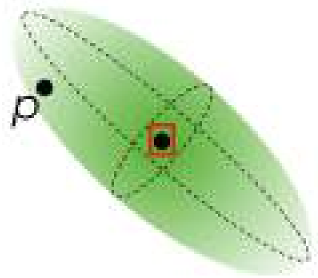
$$f_i(p) = \sigma(\alpha_i)\exp(-\frac{1}{2}(p - \boxed{\mu_i})\Sigma_i^{-1}(p - \boxed{\mu_i}))$$



Initialization

Get point cloud / random init

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For every point

Place a 3D gaussian centered on it

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For every point

Place a 3D gaussian centered on it

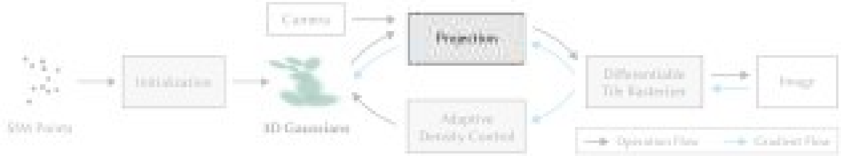
Only at initialization, isotropic gaussians

Initialization

Initialization

Initialization

Projection



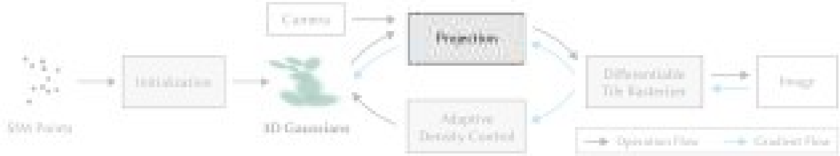
Set of 3D gaussian

Projection

Set of 3D gaussian

Project mean and covariance into 2D plane..

Projection



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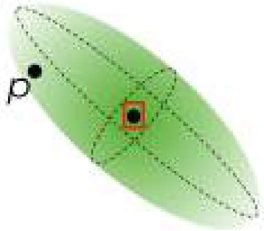

..easy to compute impact to a certain pixel

Rendering: NeRF?

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Output from points on same ray

Suppose we have many

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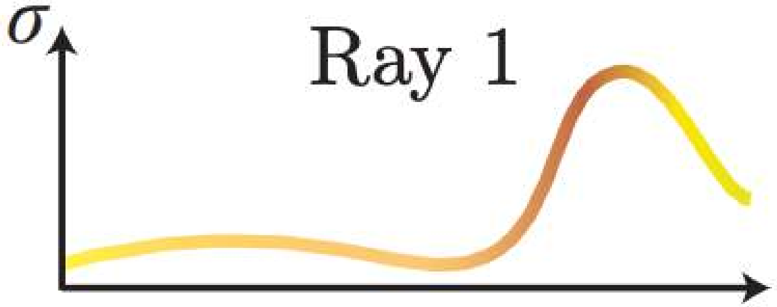
Output from points on same ray

Suppose we have many

We can produce:



Rendering: NeRF?

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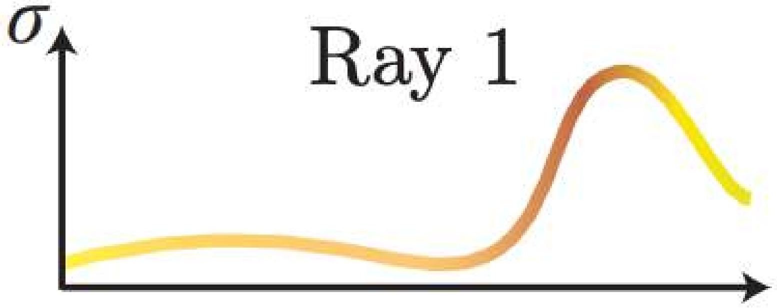

Aim: give color to whole ray

Rendering: NeRF?

Output from points on same ray

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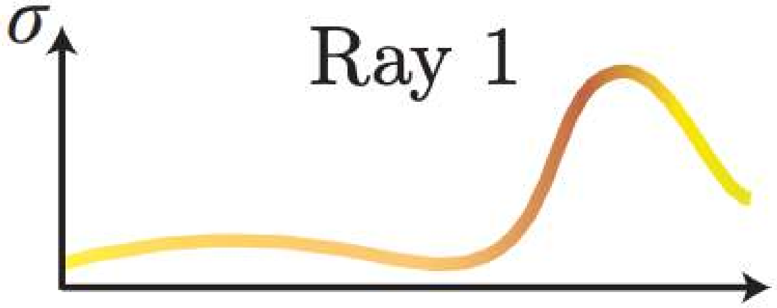
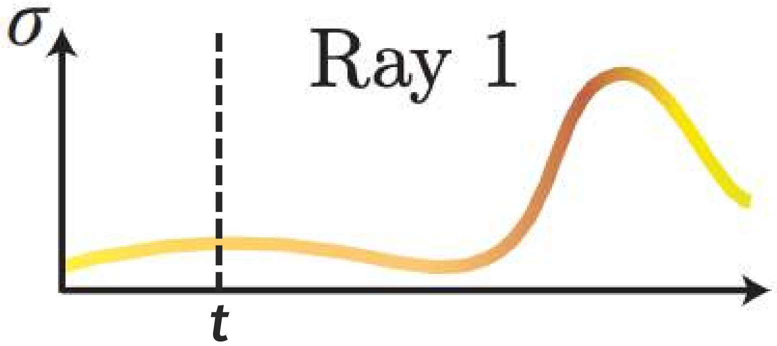


Aim: give color to whole ray
Why? determine pixel color associated to ray

Rendering: NeRF?

Point t color



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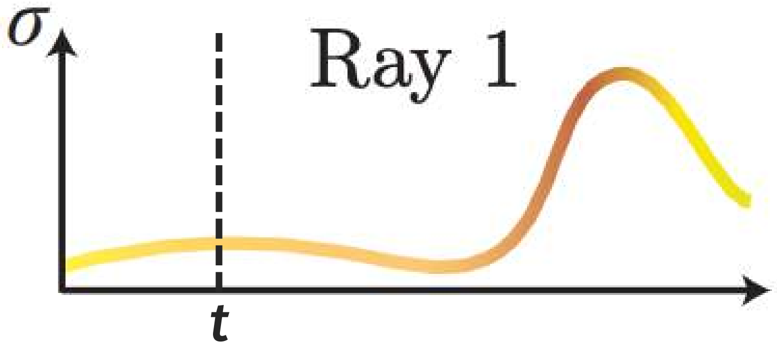
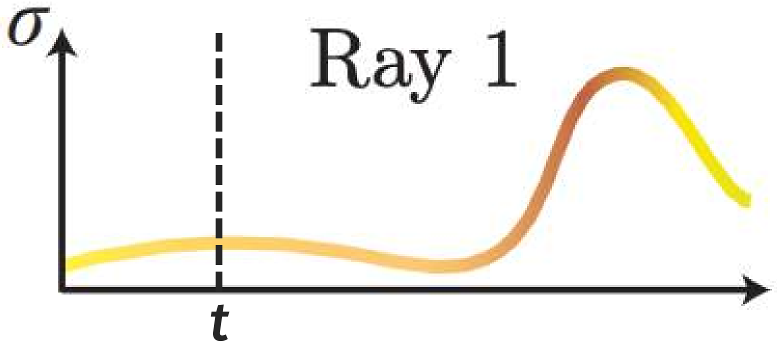


$$\mathbf{c}(\mathbf{r}(t), \mathbf{d})$$

Rendering: NeRF?

Point t color

Weighted on density (glass)



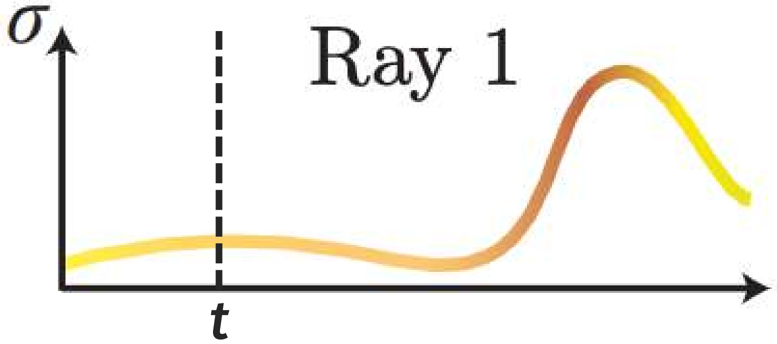
$$\sigma(\mathbf{r}(t))\mathbf{c}(\mathbf{r}(t), \mathbf{d})$$

Rendering: NeRF?

Point t color

Weighted on density (glass)

Weighted on light already stopped



$$T(t)\sigma(\mathbf{r}(t))\mathbf{c}(\mathbf{r}(t), \mathbf{d})$$

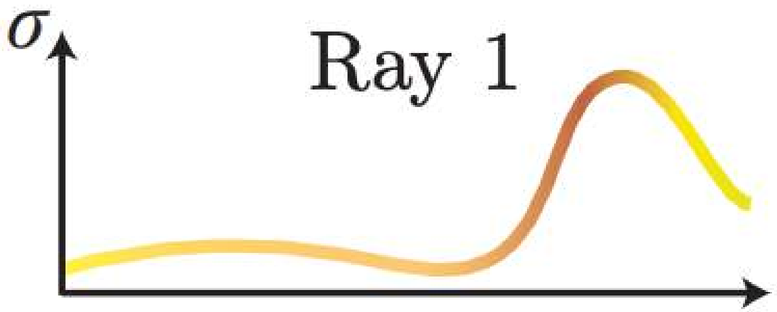
Rendering: NeRF?

Point *t* color

Weighted on density (glass)

Weighted on light already stopped

Sum of all points along ray



$$C(\mathbf{r}) = \int_{t_n}^{t_f} T(t)\sigma(\mathbf{r}(t))\mathbf{c}(\mathbf{r}(t), \mathbf{d})dt$$

Rendering: NeRF?

$$T(t) = \exp\left(-\int_{t_n}^{t} \sigma(\mathbf{r}(s))ds \right)$$

$$C(\mathbf{r}) = \int_{t_n}^{t_f} T(t)\sigma(\mathbf{r}(t))\mathbf{c}(\mathbf{r}(t), \mathbf{d})dt$$

Estimating the integral

Only 64 points per ray

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Integral finite approximation:

$$\hat{C}(\mathbf{r}) = \sum_{i=1}^{N} c_i(1 - \exp(-\sigma_i\delta_i))T_i =$$

$$\delta_i = t_{i+1} - t_i$$

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$$= \sum_{i=1}^{N} c_i \underbrace{(1 - \exp(-\sigma_i\delta_i))}_{\alpha_i} \prod_{j=1}^{i-1} \underbrace{\exp(-\sigma_j\delta_j)}_{1-\alpha_j} =$$

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Estimating the integral

Only 64 points per ray

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Comparison

$$\sum_{i=1}^{N} c_i \alpha_i \underbrace{\prod_{j=1}^{i-1} (1 - \alpha_j)}_{transmittance}$$

NeRF

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$$\sum_{i=1}^{N} c_i \alpha_i \underbrace{\prod_{j=1}^{i-1} (1 - \alpha_j)}_{transmittance}$$

$$\sum_{i \in N} c_i f_i^{2D}(p) \underbrace{\prod_{j=1}^{i-1} (1 - f_j^{2D}(p))}_{transmittance}$$

NeRF **Gaussian Splatting**

Efficiency

$$\sum_{i \in N} c_i f_i^{2D}(p) \underbrace{\prod_{j=1}^{i-1} (1 - f_j^{2D}(p))}_{transmittance}$$

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f_i^{2D} **2D Gaussian projection**

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f_i^{2D} 2D Gaussian projection

Faster?

- No NN inference for every point   #pixels · #points

Efficiency

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f_i^{2D} 2D Gaussian projection

Faster?

- No NN inference for every point #pixels · #points

- Every gaussian, one projection across pixels

Sorting algorithm

Sort gaussian by depth:

determine transmittance

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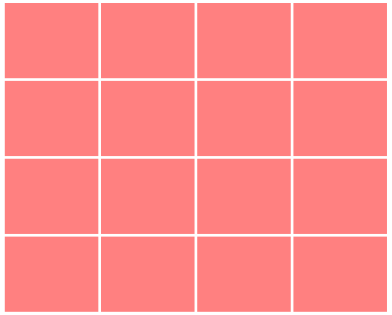
Sort gaussian by depth:

determine transmittance

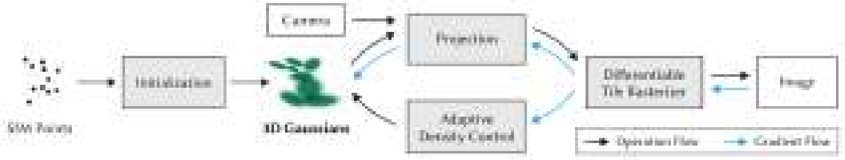
2D view:

tiled in 4x4 grid

parallelization



How does it look like?



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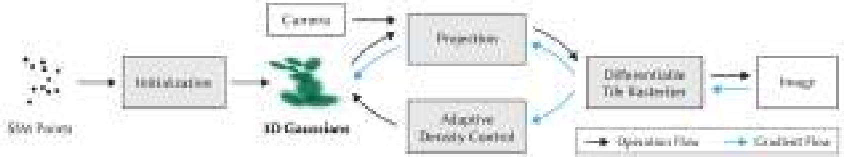

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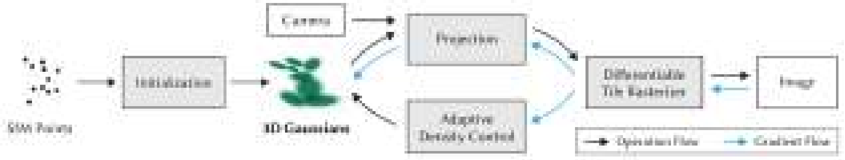

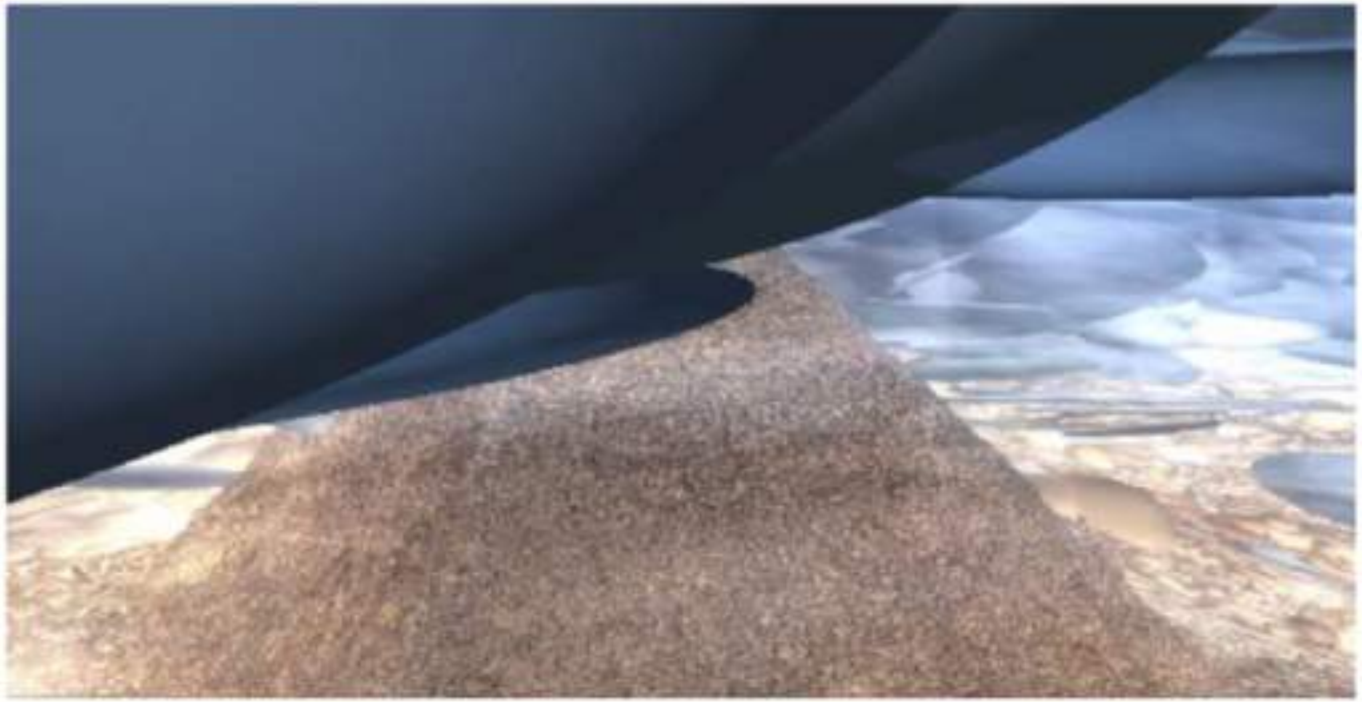

$$\mathcal{L} = (1 - \lambda)\mathcal{L}_1 + \lambda\mathcal{L}_{\text{D-SSIM}}$$

$$\lambda = 0.2$$

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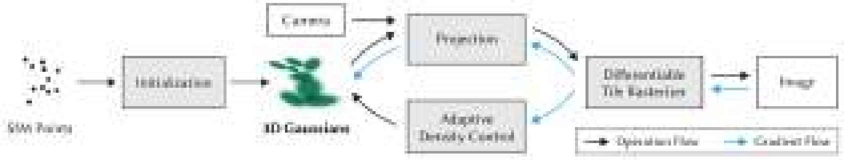

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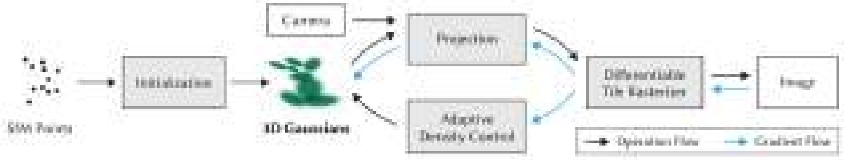
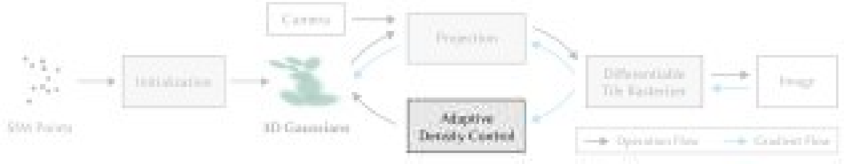
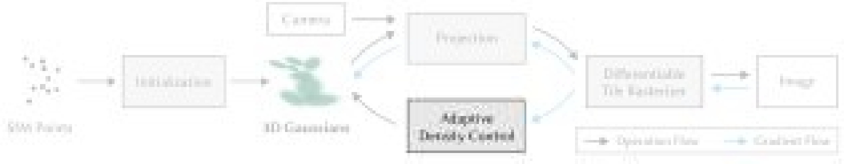
Flexibility?



Flexibility?

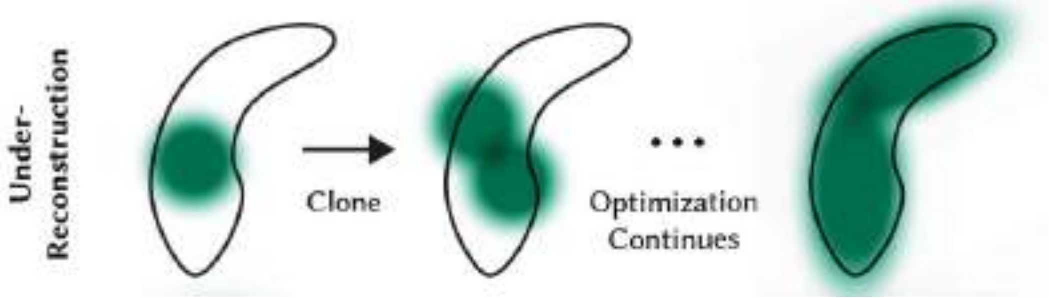


Flexibility?

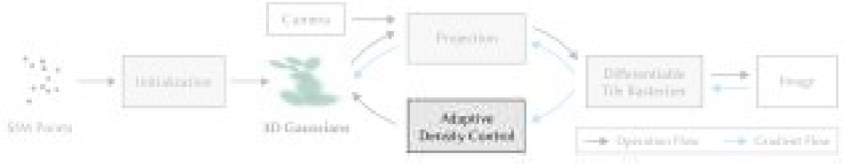


Every 100 iterations:

Under-Reconstruction check



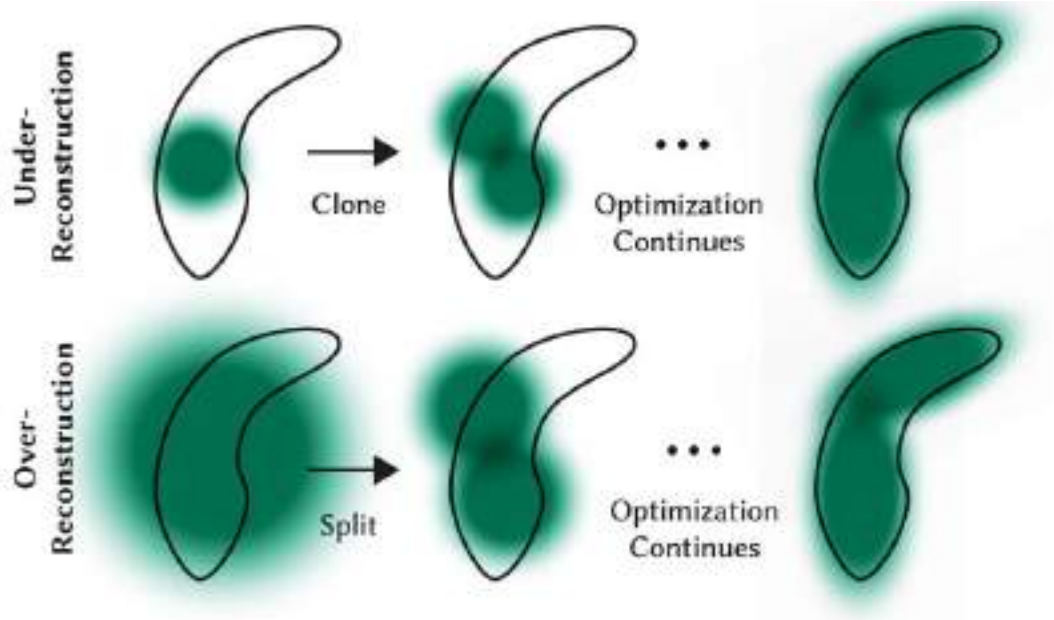
Flexibility?



Every 100 iterations:

Under-Reconstruction check

Over-Reconstruction check



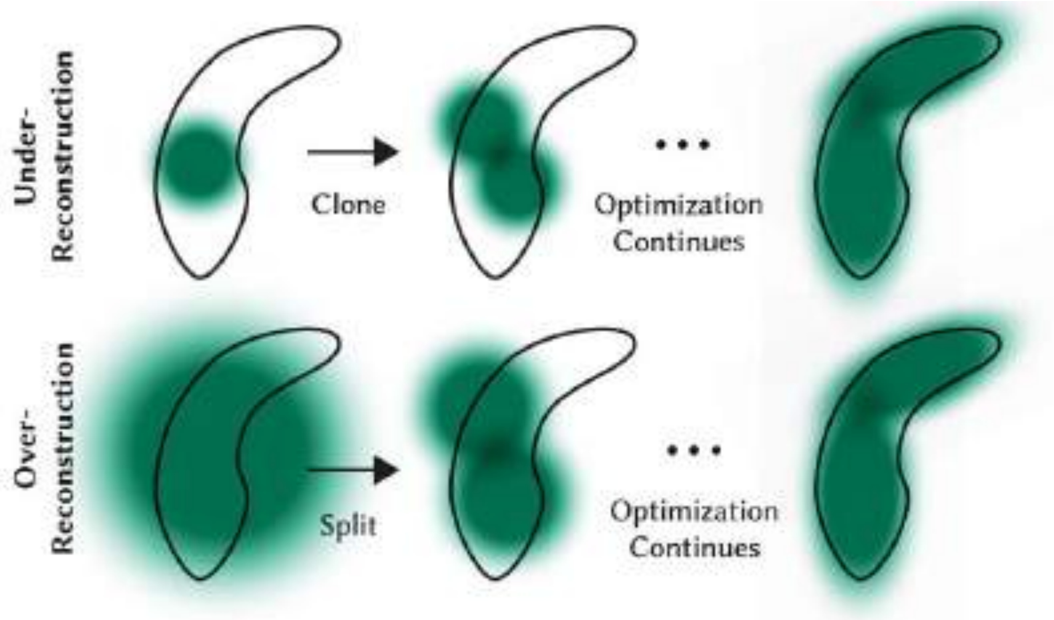
Flexibility?

Every 100 iterations:

Under-Reconstruction check

Over-Reconstruction check

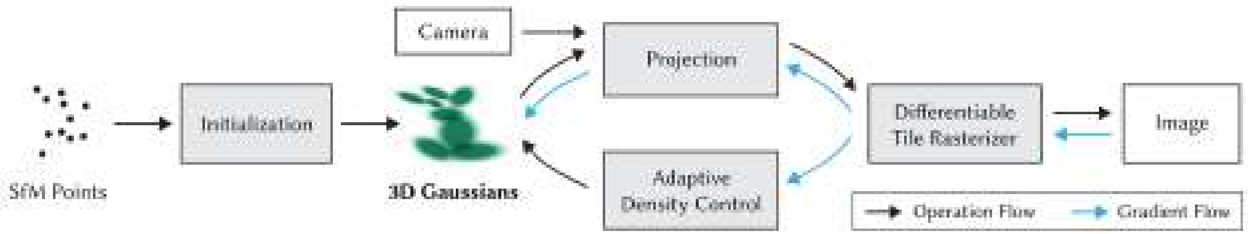
Alpha check



Recap

Algorithm 1 Optimization and Densification
w, h: width and height of the training images

$M \leftarrow$ SfM Points ▷ Positions
$S, C, A \leftarrow$ InitAttributes() ▷ Covariances, Colors, Opacities
$i \leftarrow 0$ ▷ Iteration Count
while not converged **do**
  $V, \hat{I} \leftarrow$ SampleTrainingView() ▷ Camera V and Image
  $I \leftarrow$ Rasterize(M, S, C, A, V) ▷ Alg. 2
  $L \leftarrow Loss(I, \hat{I})$ ▷ Loss
  $M, S, C, A \leftarrow$ Adam(∇L) ▷ Backprop & Step
  if IsRefinementIteration(i) **then**
    for all Gaussians (μ, Σ, c, α) **in** (M, S, C, A) **do**
      if $\alpha < \epsilon$ or IsTooLarge(μ, Σ) **then** ▷ Pruning
        RemoveGaussian()
      end if
      if $\nabla_p L > \tau_p$ **then** ▷ Densification
        if $\|S\| > \tau_S$ **then** ▷ Over-reconstruction
          SplitGaussian(μ, Σ, c, α)
        else ▷ Under-reconstruction
          CloneGaussian(μ, Σ, c, α)
        end if
      end if
    end for
  end if
  $i \leftarrow i + 1$
end while



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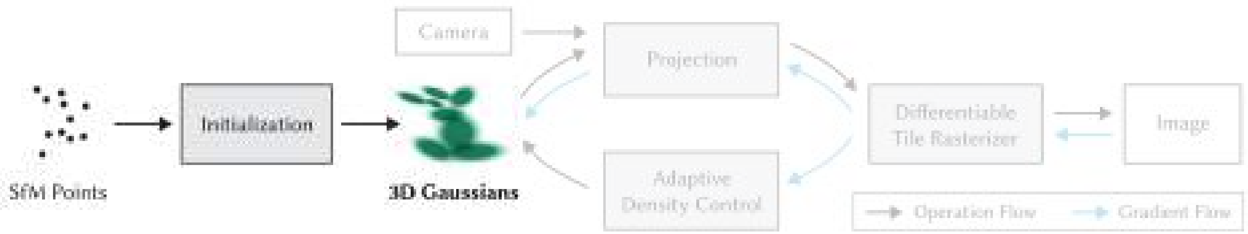

Recap

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w, h: width and height of the training images

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M ← SfM Points                                   ▷ Positions
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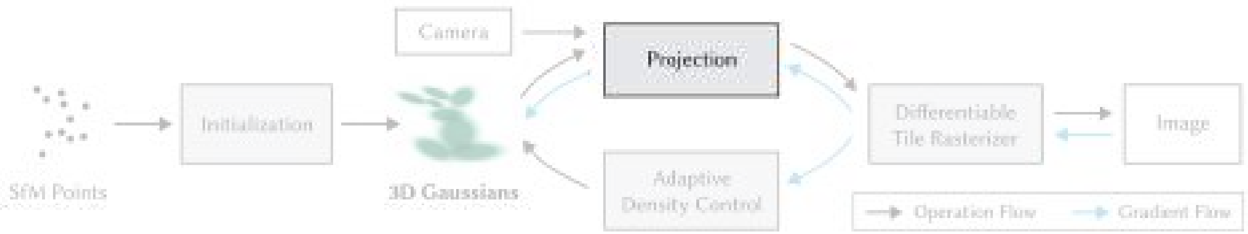

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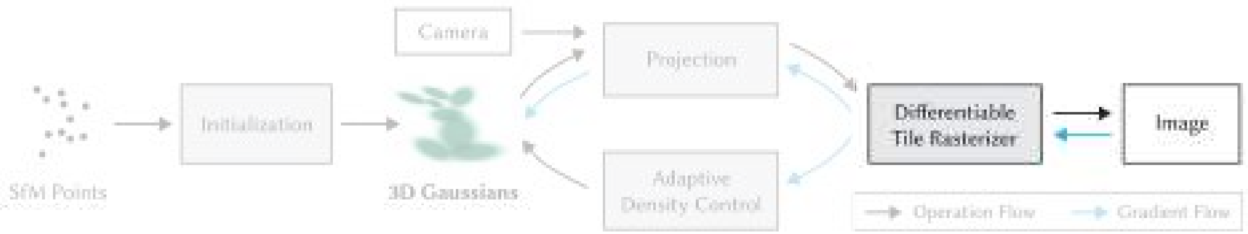

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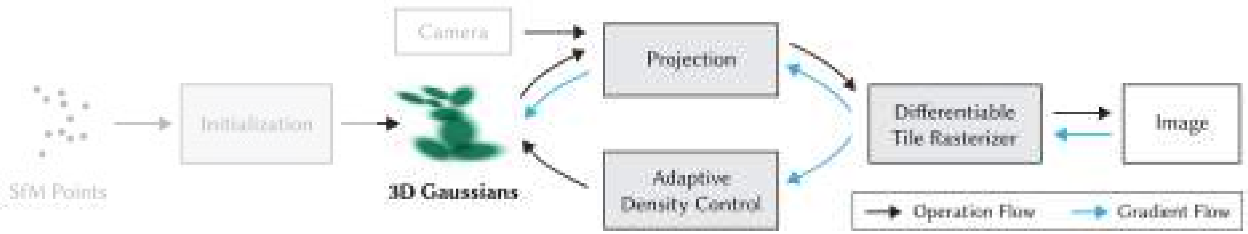

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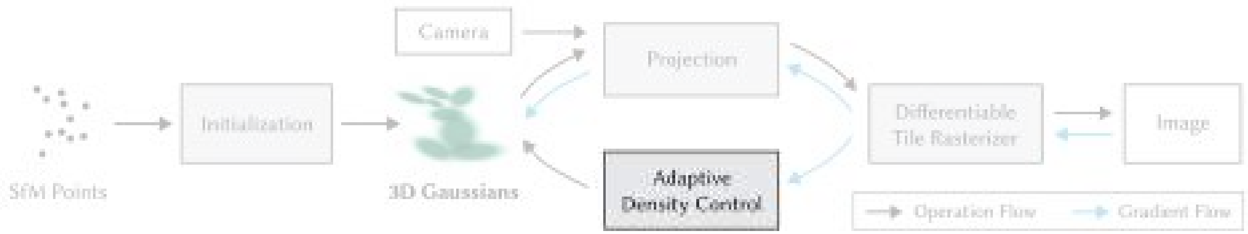

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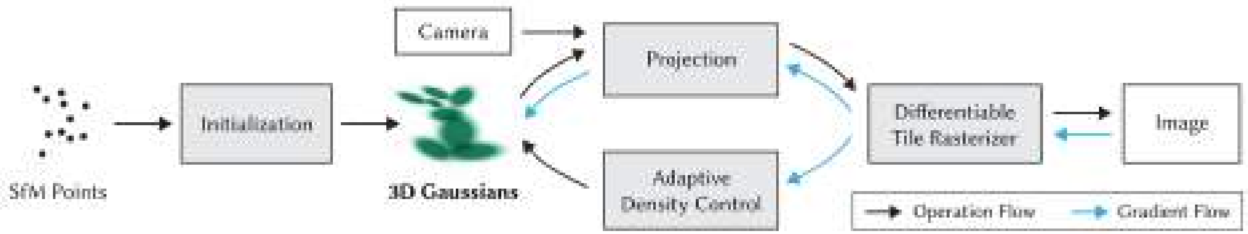
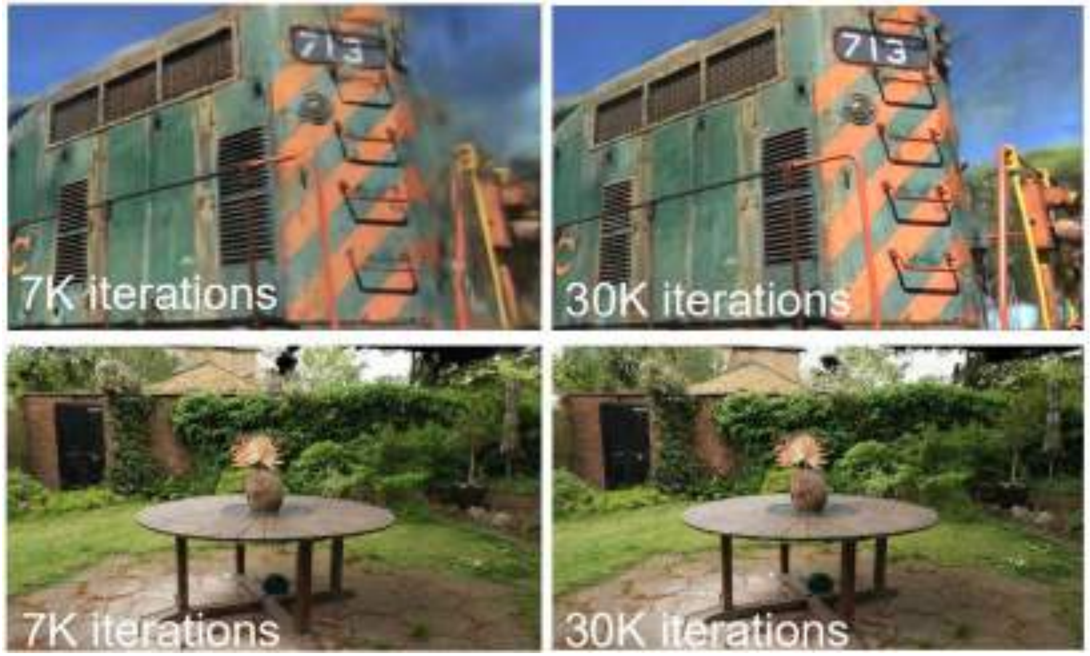
```



Quantitative results



Quantitative results

Dataset Method|Metric	Mip-NeRF360 $SSIM^{\uparrow}$	$PSNR^{\uparrow}$	$LPIPS^{\downarrow}$	Train	FPS	Mem
Plenoxels	0.626	23.08	0.463	25m49s	6.79	2.1GB
INGP-Base	0.671	25.30	0.371	5m37s	11.7	13MB
INGP-Big	0.699	25.59	0.331	7m30s	9.43	48MB
M-NeRF360	$0.792^{\dagger}$	$27.69^{\dagger}$	$0.237^{\dagger}$	48h	0.06	8.6MB
Ours-7K	0.770	25.60	0.279	6m25s	160	523MB
Ours-30K	0.815	27.21	0.214	41m33s	134	734MB

Quantitative results

Dataset	Tanks&Temples					
Method|Metric	$SSIM^{\uparrow}$	$PSNR^{\uparrow}$	$LPIPS^{\downarrow}$	Train	FPS	Mem
Plenoxels	0.719	21.08	0.379	25m5s	13.0	2.3GB
INGP-Base	0.723	21.72	0.330	5m26s	17.1	13MB
INGP-Big	0.745	21.92	0.305	6m59s	14.4	48MB
M-NeRF360	0.759	22.22	0.257	48h	0.14	8.6MB
Ours-7K	0.767	21.20	0.280	6m55s	197	270MB
Ours-30K	0.841	23.14	0.183	26m54s	154	411MB

Quantitative results

Dataset	Deep Blending					
Method|Metric	$SSIM^{\uparrow}$	$PSNR^{\uparrow}$	$LPIPS^{\downarrow}$	Train	FPS	Mem
Plenoxels	0.795	23.06	0.510	27m49s	11.2	2.7GB
INGP-Base	0.797	23.62	0.423	6m31s	3.26	13MB
INGP-Big	0.817	24.96	0.390	8m	2.79	48MB
M-NeRF360	0.901	29.40	0.245	48h	0.09	8.6MB
Ours-7K	0.875	27.78	0.317	4m35s	172	386MB
Ours-30K	0.903	29.41	0.243	36m2s	137	676MB

Qualitative results

Qualitative results

Qualitative results

UniGe | MaLGa

NeRF recap

Gaussian splatting

Conclusions

Conclusions

Gaussian splatting:

Conclusions

Gaussian splatting:

- Old meets new

Conclusions

Gaussian splatting:

- Old meets new

- SOTA results + Efficiency

Conclusions

Gaussian splatting:

- Old meets new
- SOTA results + Efficiency
- Interpretable

UniGe

MaLGa